Chapter 10.2: Use Combinations and the Binomial Theorem

A combination is a selection of r objects from a group of n objects where order doesnt matter. (such order as 1st, 2nd, 3rd, etc.)

$$_nC_r = \frac{n!}{r!(n-r)!}$$

Multiple Events: When finding the number of ways Event A and B occur you multiple. When finding the number of ways Event A or B occur you add.

A standard deck of 52 cards has 4 suits with 13 different cards in each suit.

If the order in which the cards are dealt is not important, how many different 5-card hands are possible?

$_{52}C_5 = 2{,}598{,}960$

In how many 5-card hands are all 5 cards of the same color?

color + 5cards

$_2C_1 \cdot {}_{26}C_5$

131,560

William Shakespeare wrote 38 plays that can be divided into three genres. Of the 38 plays, 18 are comedies, 10 are histories and 10 are tragedies.

How many different sets of exactly 2 comedies and 1 tragedy can you read?

$_{18}C_2 \cdot {}_{10}C_1 = 1530$

How many different sets of at most 3 plays can you read?

$_{38}C_0 + {}_{38}C_1 + {}_{38}C_2 + {}_{38}C_3 = 9178$

$1 + 38 + 703 + ? = 9178$

Counting Problems that involve "at least" or "at most" phrases are sometimes easier to solve by subtracting the possibilities you dont want.

During the school year, the girl's basketball team is scheduled to play 12 home games. You want to attend at least 3 of the games. How many different combinations of the games can you attend.

$_{12}C_3 + {}_{12}C_4 + {}_{12}C_5 + \ldots + {}_{12}C_{12}$

$2^{12} - ({}_{12}C_0 + {}_{12}C_1 + {}_{12}C_2)$

4017

Pascal's Triangle:

$$
\begin{array}{c}
\binom{0}{0} \\
\binom{1}{0} \quad \binom{1}{1} \\
\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2} \\
\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3} \\
\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}
\end{array}
$$

ow		Sur
0	1	1
1	1 1	2
2	1 2 1	4
3	1 3 3 1	8
4	1 4 6 4 1	16
5	1 5 10 10 5 1	32
6	1 6 15 20 15 6 1	64
7	1 7 21 35 35 21 7 1	128
8	1 8 28 56 70 56 28 8 1	256
9	1 9 36 84 126 126 84 36 9 1	512
10	1 10 45 124 210 252 210 124 45 10 1	1,02

The 6 members of a Model UN club must choose 2 representatives to attend a state convention. Use Pascal's triangle to find the number of combinations of 2 members that can be chosen as representatives.

Binomial Expansion:

The use of Pascal's Triangle to multiply binomials

$$(a+b)^n = {}_nC_0a^nb^0 + {}_nC_1a^{n-1}b^1 + {}_nC_2a^{n-2}b^2 + \ldots + {}_nC_na^0b^n$$

$$(a+b)^0 = 1$$
$$(a+b)^1 = a+b$$
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$$
$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Use the binomial expansion theorem

$$(x^2+y)^3$$

$(x+2)(x+2)(x+2)(x+2)(x+2)$

$(x^2+12)^{30}$

$(x+2)^5 =$

$1x^5 2^0 + 5x^4 2^1 + 10x^3 2^2 + 10x^2 2^3 + 5x^1 2^4 + 1x^0 2^5$

$$(a-2b)^4$$

$\boxed{x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32}$

Find the coefficient of x^4 in the expansion of $(3x+2)^{10}$

Homework: Chapter 10.2 pg. 694 #'s 4,8,14,16,18,22,30,34,38,50